Single frequency microwave cloaking and subwavelength imaging with curved wired media

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Abstract: We consider the cloaking properties of electromagnetic wired media deduced from arbitrary coordinate transformations. We propose an interpretation of invisibility via sub-wavelength imaging features. The quality of cloaking is assessed by the level of deformation of the image of a P-shaped source through the stretched wired media: the lesser the image deformation, the more effective the cloaking. We numerically and experimentally demonstrate a tetrahedral wired cloak with longer edge length about 7cm at a frequency of 1GHz (the cloak is thus subwavelength). The wired cloak has two functionalities: it can serve as a high-resolution imaging system over long distances, and it can also perform space transformations such as, but not limited to, cloaking at a single operation frequency.

OCIS codes: (160.1190) Anisotropic optical materials; (050.1755) Computational electromagnetic methods; (160.3918) Metamaterials; (230.3205) Invisibility cloaks.

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1. Introduction

Following the advent of invisibility cloaks and transformation optics in the last decade [1–24], advances in electromagnetic metamaterial technology have led to unprecedented flexibility in engineering structured media with highly non trivial specifications, including an independent control of the permittivity and permeability tensors with entries which can have both positive and negative values (for instance upon resonance of split ring resonators one can sculpt the permeability tensor nearly *ad libitum* for cloaking purpose [5]), or display some strong anisotropy (e.g. wired media with deep sub-wavelength imaging features [25]), or even achieve a permittivity tensor with near zero eigenvalues (for ultra-directive emission [26]). These metamaterials have enlarged the scope of functionalities of electromagnetic devices, but their design often requires complex methodologies (such as retrieval methods enabling the numerical computation of effective parameters [27] in order to work out the optimal shape of split ring resonators etc.). This complexity in the design of the structural elements of the metamaterials refrains the applicability of transformation optics (TO), which was introduced so as to provide an intuitive and direct mathematical tool to design optical devices with novel functionalities. The philosophy of TO is that a designer envisions a fictitious space with some topological features (such as a hole, or a space folding) that enact a desired electromagnetic phenomenon (such as invisibility [4], or perfect lensing [28]), and the transformation method yields, in a direct way, the complex material property specification (more precisely the inhomogeneous anisotropic tensors of permittivity and permeability) that implements the desired electromagnetic response. However, the strong anisotropy required by cloaking systems makes them difficult to fabricate, and there seems to be a need for simplification here. In this article, we propose to relax the constraints on material parameters (we just work on the tensor of permittivity i.e. we focus on the control of the electric field) in order to achieve some simple design of transformed media using curved conducting wires. Thin straight wires have been known since the work of Belov et al. [25] to allow for deep sub-wavelength imaging features through infinite anisotropy of the permittivity tensor describing the so-obtained effective medium. However, one could go one step further and use such an extreme anisotropy to mold electromagnetic field propagation in curved wired media. We will show that one can for instance create a hole in space, of any desired shape, in the present case a tetrahedron, see Fig. 1, which disturbs only very moderately the imaging process of straight wired media. We stress that our numerical and experimental results at microwave frequencies can be easily reproduced with very little technology at hand. Our design might also inspire further work at optical frequencies with resonant dielectric structures, or lead to similar control of other types of waves (such as acoustic).

2. On the design of transformed wired effective media

The arbitrarily shaped cloaks, which we would like to design, require a coordinate transformation. This transformation compresses all the space in a given volume, say a tetrahedron, a cube or a sphere, into the same region with a hole inside (say a hole which is a smaller tetrahedron, but it could be a cube, a sphere or any other geometric shape). Mathematically, this means we consider a position vector **x** in the original coordinate system (Fig. 2(a)), where it has components x_i, and we compute the corresponding position vector **x**' in the transformed coordinate system (Fig. 2(b)), where it has components x_i'. We stress that the magnitude of the position vector r = |**x**|, is independent of the coordinate system since $r=\left(x^i x^j \delta_{ij}\right)^{1/2}=\left(x^{i'} x^{j'} \delta_{i'j'}\right)^{1/2}$ $r'=\left(x^{i'} x^{j'} \delta_{i'j'}\right)^{1/2}$ where $\delta_{i'j'}$ is the Kronecker symbol. From the TO standpoint, or in the effective materials interpretation as we shall see with the next formula, the straight wired medium (Fig. 2(a)) should be equivalent to the curved wired media (Fig. 2(b)). This means the hole created in the wired medium should not alter the electromagnetic field i.e. invisibility should be at work. Importantly, the geodesics of the transformed medium correspond in our approach to the curved medium design. This means that to design the shape of the curved wires, we need only consider the components x_i', to be the components of a Cartesian vector, and its magnitude, which we will call r′, is found using the appropriate flat space metric.

On the other hand, the wired medium works as an infinite anisotropic effective medium such that

$$\begin{pmatrix} k_0^2\varepsilon_{xx}-k_z^2-k_y^2 & k_0^2\varepsilon_{xy}+k_xk_y & k_0^2\varepsilon_{xz}+k_xk_z \\ k_0^2\varepsilon_{yx}+k_xk_y & k_0^2\varepsilon_{yy}-k_z^2-k_y^2 & k_0^2\varepsilon_{yz}+k_yk_z \\ k_0^2\varepsilon_{zx}+k_xk_z & k_0^2\varepsilon_{zy}+k_yk_z & k_0^2\varepsilon_{zz}-k_z^2-k_y^2 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

where $\varepsilon_{zz}=\varepsilon_0\left(1-\dfrac{k_p^2}{k_0^2-q^2}\right)$ and $k_0=\omega/c$ is the wavenumber in vacuum (defined by the ratio of angular frequency to light velocity), k_p is the wavenumber of the plasma frequency and q is the component of the Bloch vector along the wires. Clearly, upon resonance the dispersion characteristics of the effective medium are ruled by $(k_0^2\varepsilon_{zz}-k_z^2-k_y^2)E_z=0$ along the wires' main axis.

#229291 - $15.00 USD Received 2 Jan 2015; accepted 7 Apr 2015; published 14 Apr 2015

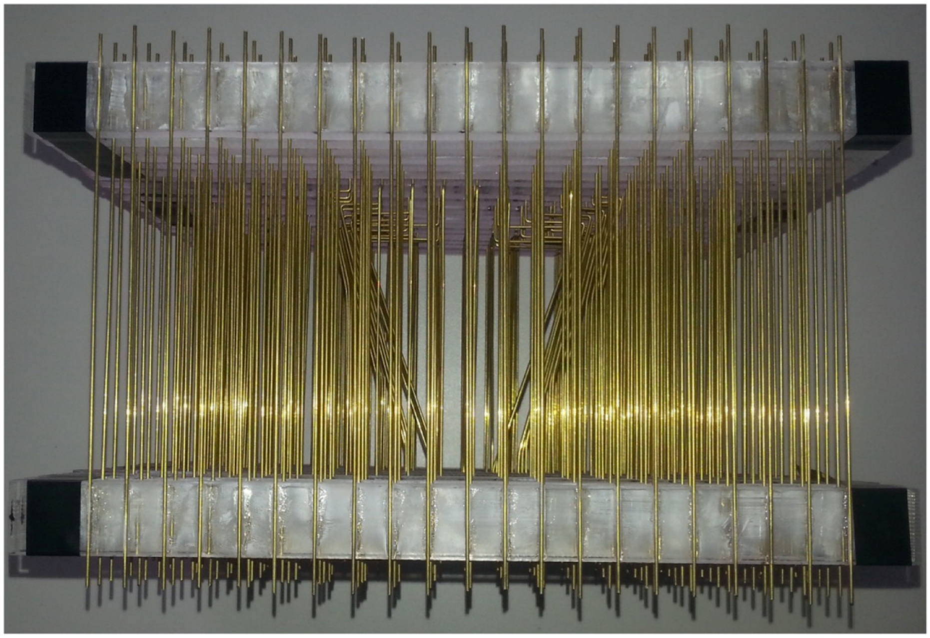

Fig. 1. Photo of the fabricated wired medium with a tetrahedral cloak. It consists of 440 brass wires of radius 0.5mm that connect two interfaces (made of Plexiglas) structured with square arrays of pitch 1cm which are located 15cm apart.

The specific entries of the effective anisotropic tensor of permittivity could be deduced from a retrieval method [27], but the essential feature of the system is that the distribution of the field on a front interface of the array of wires should be identical to that on the back interface [25,28]. Importantly, a similar effective anisotropic tensor of permittivity could be achieved via dielectric fibres under certain oblique incidence [29–31], which suggests that one could replicate the study at optical wavelengths. In what follows, we shall focus on such wired media in order to design an invisibility cloak by revolution about the z-axis [32]. We stress that our cloak will be constrained to a single frequency operation and to an electric line current source as the analysis heavily relies upon the plasma frequency of the electric resonant mode in the array of conducting wires. However, one could envisage other designs such as fisheye nets which would ensure not only near zero permittivity but also near zero permeability upon plasma resonance [33], for enhanced control of electric and magnetic fields.

3. Numerical analysis of various curved wired media

In order to verify the concept described above, numerical simulations of the metallic wired media presented in Figs. 2(a) and 2(b) were performed using the CST Microwave Studio package, see Fig. 2(c)-(f).

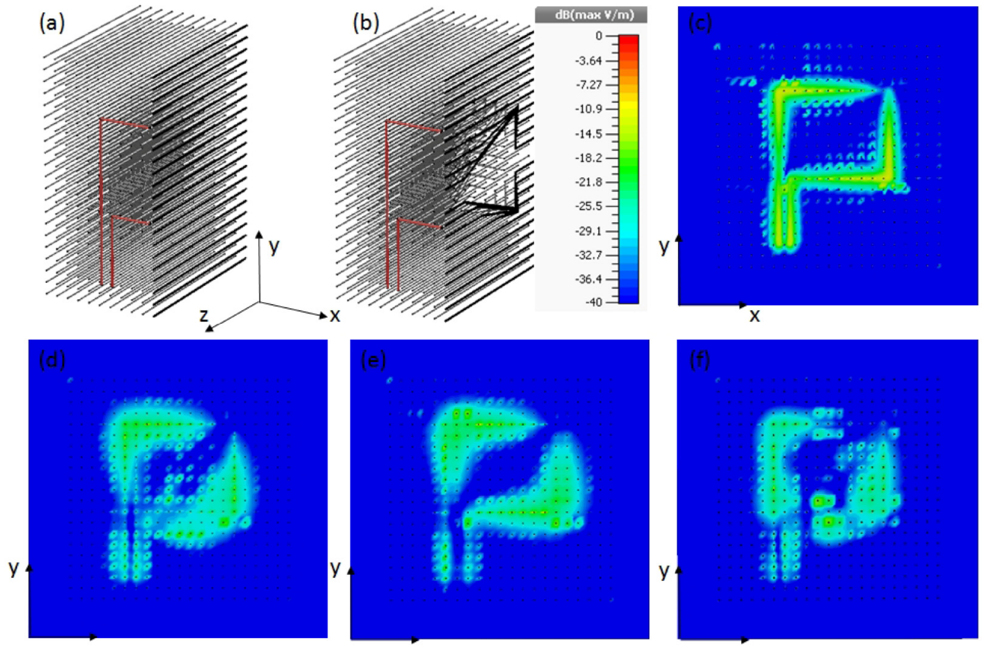

Fig. 2. Schematics and numerics of metallic wired media for cloaking of a P shape electric line current source at 1GHz: Straight wired medium originally proposed in [25] (a) and curved wire medium with a tetrahedral cloak whose inner and outer boundaries are defined by two pyramids of respective heights 8.1cm and 12cm along z and edge lengths 1.4cm and 5.4cm along x and y (b); Color maps of the modulus of the total electric field in the source plane (c) and the image plane for a straight wired medium (d), a curved wired medium with a tetrahedral cloak (e) and a curved wired medium with a tetrahedral cloak surrounding a cuboid (4x4x4.4cm^3) metallic obstacle (f). The strong similarities between fields in (d-f) are noted. Color scale is in decibels.

The lens consisting of an array of 21*21 brass wires (up to the removal of the center wire) excited by a source in the form of a letter P was modeled. The operating frequency *f* is 1 GHz, the length of the wires' thickness of the slab *d* is 15 cm. d corresponds to a half wavelength in the free space, the period of the lattice *a* is 1 cm (so a thirtieth of the wavelength), the radius of the brass wires *b* is 0.5 mm. The source in the form of a letter P is placed at a distance *c* = 5 mm from the front interface of the wired medium and fed by an electric current. Results of the CST simulations are presented in Fig. 2 and Fig. 3, for straight (Fig. 2(c) & Fig. 3(a)) and curved (Fig. 2(d) and 2(e) & Fig. 3(b) and 3(c)) wired media, respectively. The source produces the deep subwavelength distribution of the electric field at the front interface of the slab, see Fig. 2(c) (which is identical for straight and curved wires). Regarding the fabricated curved wired medium shown in Fig. 1, the *p*-polarized contribution of the field is canalized from the front to the back interfaces of the wired medium and it forms an image, see Fig. 2(d) (straight wires), Fig. 2(e) (curved wires) and Fig. 2(f) (curved wires and metallic obstacle). The front and back distributions of the electric field are very similar in the straight and curved wired media, even when there is an obstacle, which demonstrates the invisibility feature of cloaking.

Another important feature of cloaking is wave protection, whereby the central region within the curved wired medium should display less electric field than elsewhere. This can be seen in Fig. 3, where the distribution of electric field has been numerically computed in two planes passing through the central region in the straight, Fig. 3(a), and curved, Fig. 3(b) and 3(c), wired media without (b) and with (c) an metallic obstacle. The experimental result shown in Fig. 3(d) for the curved wired medium without obstacle is clearly similar to Fig.

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3(b). It is noteworthy that we numerically compared the transformed effective permittivity tensors that describe inhomogeneous and anisotropic spaces with a cubical, spherical and tetrahedral cloak. The latter turned out to have optimal cloaking property, since the image appears to be less deformed than for spherical and cubical cloaks. Moreover, we numerically observe that the electromagnetic field is the lowest in the invisibility region formed by the wired cloak, when it has a spherical shape. We conclude that the smoother the cloak's boundary, the better the wave protection. In passing, the single operation frequency could become multi-frequency with a similar approach to [34].

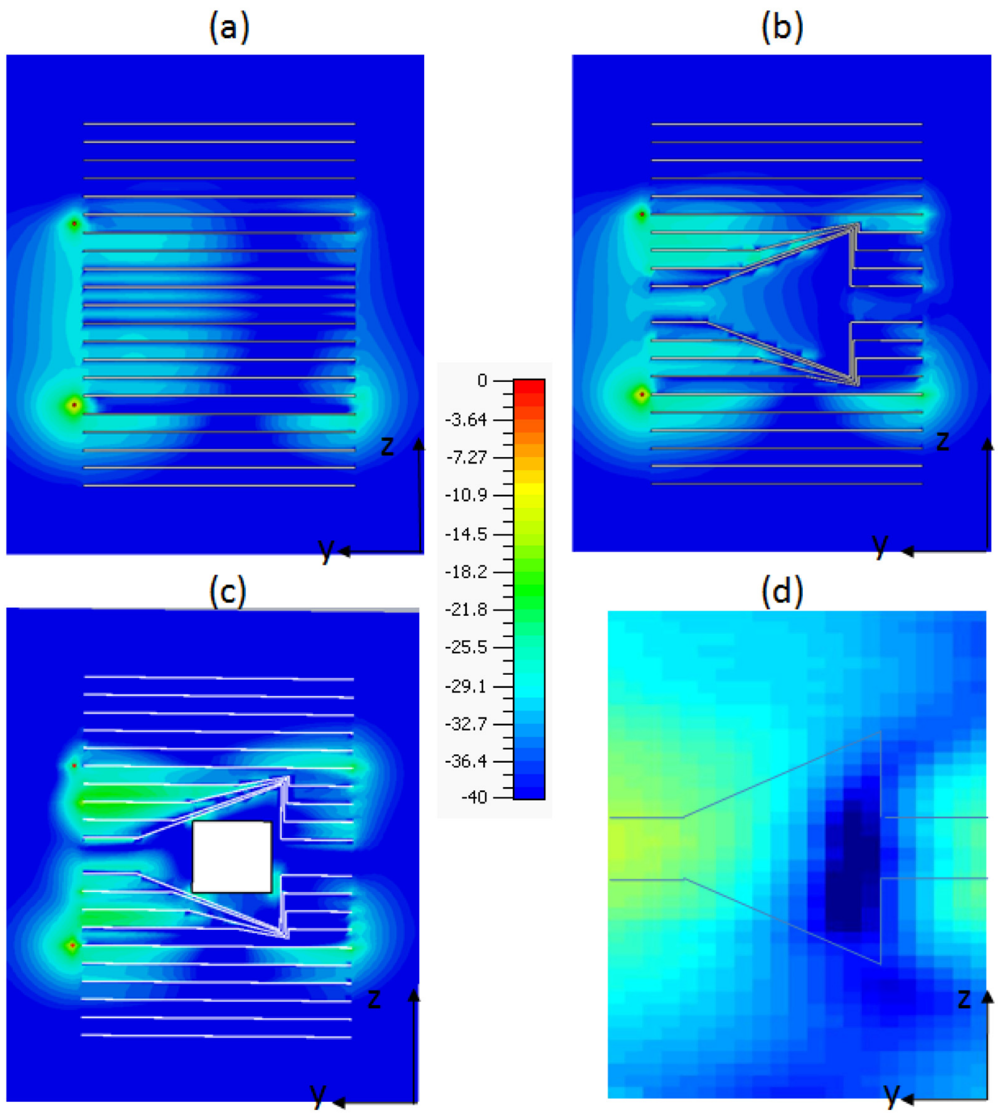

Fig. 3. Numerical and experimental results for cloaking versus protection of a P shape electric line current source at frequency 1GHz through a straight wired medium (a), a curved wired medium with a tetrahedral cloak (b) and a curved wired medium with a tetrahedral cloak surrounding a cuboid metallic obstacle (c) as in Fig. 2; (d) shows the experimental counterpart of (b); Color maps show the modulus of the total electric field in the xy-plane passing through the center of the wired medium. Color scales are in decibels and have the same bounds as in Fig. 4 for comparison with numerical (a)-(c) and experimental (d) maps. The reduction of total electric field inside the tetrahedron cloak in (c-d) is noted. Color scale is in decibels (dB).

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4. Experimental proof of concept for tetrahedron wired cloak

We show in Fig. 4 the distribution of the total electric field experimentally achieved for the tetrahedron cloak, which is conceived via transformation optics tools for cloaks with a revolution axis [32]. The quality of experimental versus numerical imaging results can be assessed in Figs. 4(a) and 4(b) and Figs. 4(c) and 4(d), respectively, where absolute values of electric field in the vicinity of the back interface are plotted in dB color scale. In order to confirm cloaking, we add an obstacle within the cloak in Figs. 4(b) and 4(d), which compares well with the cloak without obstacle in Figs. 4(a) and 4(c). The resolution of the imaging system can be evaluated as a radius of spot at a half of field intensity level. We found that the resolution is equal to 2 cm (double period of the structure) as in [25] for the straight wired medium, which is one-fifteenth of the wavelength, and a similar resolution is achieved for the curved wired medium.

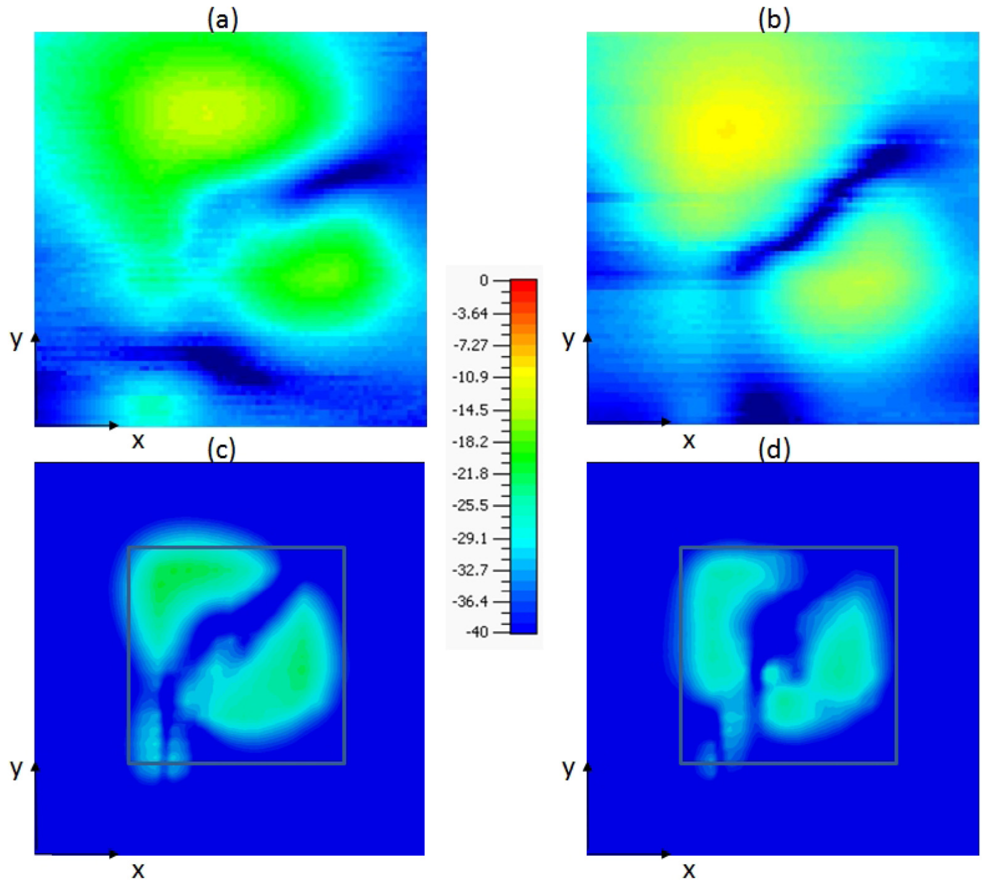

Fig. 4. Experimental ((a) and (b)) versus numerical ((c) and (d)) results for lensing of a P shape electric line current source through the curved wired medium (back end) without (a) and (c) and with (b) and (d) the metallic cuboid obstacle within the cloak. The marked area in (c) and (d) corresponds to the map in (a) and (b). The strong similarities between all four panels are noted. Color scale is in decibels (dB).

5. Conclusion

In conclusion, we have numerically and experimentally demonstrated invisibility features of curved wired media for microwaves at a single frequency corresponding to the plasma resonance of the corresponding effective media. The wired media facilitate the design of arbitrarily shaped cloaks, which are relatively easy to fabricate since they merely consist of curved conducting wires shaped as the geodesics of the corresponding transformed media. The cloaks so designed offer some form of microwave protection for an object placed inside the invisibility region (further numerical simulations suggest protection is all the more efficient that the cloak's boundary is smooth). We hope our approach of transformed media with curved wires will foster experimental efforts in cloaks, concentrators, rotators for single frequency operation for microwave, and optical, wavelengths.

Acknowledgments

R.A., S.E., and S.G. are thankful for a PEPS INSIS/CNRS funding through project CLOAK EXTERNE. S.G. acknowledges funding from ERC through project ANAMORPHISM.